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Gauged $\mathrm{U}(1)_R$ symmetries and Fayet-Iliopoulos terms in 5D orbifold supergravity

Hiroyuki Abe

Department of Physics, Korea Advanced Institute of Science and Technology
Daejeon 305-701, Korea
E-mail: abe@hep.kaist.ac.kr

Kiwoon Choi

Department of Physics, Korea Advanced Institute of Science and Technology
Daejeon 305-701, Korea
E-mail: kchoi@hep.kaist.ac.kr

Abstract: We discuss a gauged $\mathrm{U}(1)_R$ supergravity on five-dimensional (5D) orbifold (S^1/Z_2) in which both a Z_2-even U(1) gauge field and the Z_2-odd graviphoton take part in the $\mathrm{U}(1)_R$ gauging. Based on the off-shell formulation of 5D supergravity, we analyze the structure of Fayet-Iliopoulos (FI) terms allowed in such model. Introducing a Z_2-even $\mathrm{U}(1)_R$ gauge field accompanies new bulk and boundary FI terms in addition to the known integrable boundary FI term which could be present in the absence of any gauged $\mathrm{U}(1)_R$ symmetry. Some physical consequences of these new FI terms are examined.

Keywords: Field Theories in Higher Dimensions, Supergravity Models.

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Contents

1. Introduction

Supersymmetry (SUSY) is an attractive candidate for physics beyond the standard model (SM) as it can stabilize the electroweak scale against the high energy radiative corrections to the Higgs boson mass. Furthermore SUSY is a fundamental ingredient of the only known consistent theory of quantum gravity, i.e., superstring theory. To be phenomenologically viable, SUSY should be broken by a nonzero vacuum value of F and/or D auxiliary component. This can be achieved by introducing a tadpole term for the F and/or D-component in the effective lagrangian, for instance an O'Raifeartaigh term for F-breaking [1] and a Fayet-Iliopoulos (FI) term for D-breaking [2]. However in locally supersymmetric theory D-term SUSY breaking is severely limited since FI term is not allowed *unless* the corresponding U(1) is either an R-symmetry [3] or a so-called pseudo-anomalous U(1) symmetry with Green-Schwarz anomaly cancellation mechanism [4].

Recently five-dimensional (5D) supergravity (SUGRA) on the orbifold S^1/Z_2 has been studied as an interesting theoretical framework for physics beyond the SM. It has been noted that 5D orbifold SUGRA with a $\mathrm{U}(1)_R$ symmetry gauged by the Z_2-odd graviphoton can provide the supersymmetric Randall-Sundrum (RS) model [5] in which the weak to Planck scale hierarchy can arise naturally from the geometric localization of 4D graviton [6]. In this case, the bulk cosmological constant and brane tensions which are required to generate the necessary AdS_5 geometry appear in the lagrangian as a consequence of the $\mathrm{U}(1)_R$ FI term with Z_2-odd coefficient.

In this paper we consider a more generic orbifold SUGRA which contains a Z_2-even 5D gauge field A^X_μ participating in the $\mathrm{U}(1)_R$ gauging. If 4D $N = 1$ SUSY is preserved by the compactification, the 4D effective theory of such model will contain a gauged $\mathrm{U}(1)_R$ symmetry associated with the zero mode of A^X_μ, which is not the case when the 5D $\mathrm{U}(1)_R$ is gauged only through the Z_2-odd graviphoton. Based on the known off-shell formulation [7], we formulate a gauged $\mathrm{U}(1)_R$ SUGRA on S^1/Z_2 in which both A^X_μ and the graviphoton take part in the $\mathrm{U}(1)_R$ gauging and then analyze the structure of FI terms allowed in such model. As expected, introducing a Z_2-even $\mathrm{U}(1)_R$ gauge field accompanies new bulk and boundary FI terms in addition to the known integrable boundary FI term which could be present in the absence of any gauged $\mathrm{U}(1)_R$ symmetry [8, 9, 10, 11]. As we will see, those new FI terms can have interesting implications to the quasi-localization of the matter zero modes in extra dimension [12] and also to the SUSY breaking and radion stabilization.

The organization of this paper is as follows. In the next section we provide a formulation for $\mathrm{U}(1)_R$-gauged orbifold SUGRA based on the Kugo-Ohashi off-shell SUGRA formulation [7]. In section 3, we discuss the conditions for unbroken $N = 1$ SUSY for generic 4D Poincaré invariant field configurations and also some physical consequences of the FI terms of the Z_2-even $\mathrm{U}(1)_R$ gauge symmetry. Section 4 is a conclusion.

2. 5D orbifold supergravity with gauged $\mathrm{U}(1)_R$

In this section we provide a formulation for 5D SUGRA on S^1/Z_2 in which both a Z_2-even gauge field and the Z_2-odd graviphoton take part in the $\mathrm{U}(1)_R$ gauging. As a minimal example, we consider a model containing three $\mathrm{U}(1)$ vector multiplets:

$$\begin{aligned}
\mathcal{V}_Z &= \left(\alpha,\, A^Z_\mu,\, \Omega^{Zi},\, Y^{Zij}\right), \\
\mathcal{V}_X &= \left(\beta,\, A^X_\mu,\, \Omega^{Xi},\, Y^{Xij}\right), \\
\mathcal{V}_S &= \left(\gamma,\, A^S_\mu,\, \Omega^{Si},\, Y^{Sij}\right),
\end{aligned}$$

together with a compensator hypermultiplet $\mathcal{H}_c$ and physical hypermultiplets $\mathcal{H}_p$:

$$\begin{aligned}
\mathcal{H}_c &= (\mathcal{A}^x_i, \eta^x, \mathcal{F}^x_i), \\
\mathcal{H}_p &= (\Phi^x_i, \zeta^x, F^x_i),
\end{aligned}$$

where $M^A = (\alpha, \beta, \gamma)$ $(A = Z, X, S)$ are real scalar components, Ω^i_A $(i = 1, 2)$ are $\mathrm{SU}(2)_{\mathbf{U}}$-doublet symplectic Majorana spinors, $Y^{Aij} = Y^{Aji}$ are $\mathrm{SU}(2)_{\mathbf{U}}$-triplet auxiliary components, $\mathcal{A}^x_i, \Phi^x_i$ $(x = 1, 2)$ are quaternionic hyperscalars, η^x, ζ^x are symplectic Majorana hyperinos, and $\mathcal{F}^x_i, F^x_i$ are the auxiliary components of hypermultiplets. Here $\mathcal{V}_Z$ is the model-independent central charge vector multiplet which contains the Z_2-odd graviphoton A^Z_μ, $\mathcal{V}_X$ is a vector multiplet which contains a Z_2-even $\mathrm{U}(1)$ gauge field A^X_μ, and $\mathcal{V}_S$ is a non-physical vector multiplet which is introduced to obtain the necessary Z_2-odd couplings through the 4-form multiplier mechanism of ref. [13]. The Z_2-parities of these components are summarized in table 1.

Because we have a single compensator hypermultiplet $\mathcal{H}_c$, the corresponding quaternionic manifold spanned by physical hyperscalars $\mathcal{H}_p$ $(p = 1, 2, \ldots, n_H)$ is $\mathrm{USp}(2, 2n_H)$ / $\mathrm{USp}(2) \times \mathrm{USp}(2n_H)$. In the following, we will use frequently a matrix notation for hyperscalars, e.g.,

$$\Phi \equiv \begin{pmatrix} \Phi^{x=1}_{i=1} & \Phi^{x=1}_{i=2} \\ \Phi^{x=2}_{i=1} & \Phi^{x=2}_{i=2} \end{pmatrix} = \begin{pmatrix} \Phi_+ & \Phi_- \\ -\Phi^*_- & \Phi^*_+ \end{pmatrix},$$

Z_2-even	$\alpha,\, A_{\underline{\mu}}^{X,S},\, A_y^Z,\, (Y^{X,S})^{(3)},\, (Y^Z)^{(1,2)}$	$(\mathcal{A},\Phi)_{i=1}^{x=1},\, (\mathcal{A},\Phi)_{i=2}^{x=2},\, (\mathcal{F},F)_{i=2}^{x=1},\, (\mathcal{F},F)_{i=1}^{x=2}$
Z_2-odd	$\beta,\, \gamma,\, A_{\underline{\mu}}^{Z},\, A_y^{X,S},\, (Y^Z)^{(3)},\, (Y^{X,S})^{(1,2)}$	$(\mathcal{A},\Phi)_{i=2}^{x=1},\, (\mathcal{A},\Phi)_{i=1}^{x=2},\, (\mathcal{F},F)_{i=1}^{x=1},\, (\mathcal{F},F)_{i=2}^{x=2}$

Table 1: The Z_2-parity assignment of component fields. Here $\underline{\mu} = 0,1,2,3$, while y is the fifth coordinate of S^1/Z_2 and $Y_j^{Ai} = \sum_{r=1}^{3}(i\sigma_r)^i_j (Y^A)^{(r)}$ $(A = Z, X, S)$.

where $\Phi_\pm$ are Z_2 parity eigenstates, and the similar notation is adopted for the compensator $\mathcal{A}$. In this matrix notation, the symplectic reality condition and the Z_2 boundary condition are given by

$$\Phi^*(y) = i\sigma_2\Phi(y)i\sigma_2^T\,, \qquad \Phi(-y) = \sigma_3\Phi(y)\sigma_3\,. \tag{2.1}$$

We will also use a matrix notation for the $\mathrm{SU(2)_U}$-triplet fields as, e.g., $Y^A \equiv Y_j^{Ai}$. Note that the $\mathrm{SU(2)_U}$ index is raised and lowered by an antisymmetric tensor $\epsilon_{ij} = \epsilon^{ij} = i\sigma_2$ like, e.g., $Y_j^{Ai} = Y^{Aik}\epsilon_{kj} = \epsilon^{ik}Y^A_{kj}$.

The most general hypermultiplet gauging consistent with the Z_2 orbifolding and the symplectic reality condition (2.1) is given by

$$\begin{aligned}\Big(T_Z, T_X, T_S\Big)\Phi &= \Big(0,\, q,\, c\Big)i\sigma_3\Phi\,,\\ \Big(T_Z, T_X, T_S\Big)\mathcal{A} &= \Big(0,\, -r,\, -\tfrac{3}{2}k\Big)i\sigma_3\mathcal{A}\,,\end{aligned}$$

where $T_{Z,X,S}$ are the $\mathrm{U}(1)_{Z,X,S}$ generators and q, c, r, k are real constants. The vector multiplet part of 5D SUGRA is determined by the norm function $\mathcal{N}$ which is a homogeneous cubic polynomial of M^A: $\mathcal{N} = C_{ABC}M^AM^BM^C$ for a totally symmetric constants C_{ABC}. In this paper, we choose

$$\mathcal{N} = \alpha^3 - \frac{1}{2}\alpha\beta^2 + \frac{1}{2}\xi_{FI}\alpha\beta\gamma\,,$$

which corresponds to a minimal model for our purpose.

Under the constraint on $\mathcal{V}_S$ induced by the four-form multiplier field which will be introduced below, a nonzero ξ_{FI} gives an integrable boundary FI term for the $\mathrm{U}(1)_X$ vector multiplet: $-\frac{1}{2}\xi_{FI}Y^{X\,(3)}\partial_y\epsilon(y)$ where we use an isovector notation $Y^A = \sum_{r=1}^{3}(i\sigma_r)Y^{A\,(r)}$, the gauge charge k of the compensator gives a negative bulk cosmological constant $-6k^2$ for AdS_5 geometry as well as the correct integrable boundary tension $3k\partial_y\epsilon(y)$, and finally the gauge charge c of the physical hypermultiplet gives a hyperino kink mass $c\epsilon(y)$. The four-form multiplier mechanism of ref. [13] provides a dynamical way to generate Z_2-odd couplings proportional to the periodic sign-function $\epsilon(y) = y/|y|$ which obeys

$$\begin{aligned}\epsilon(y) = -\epsilon(-y) &= \epsilon(y + 2\pi R) = 1 \qquad (0 < y < \pi R)\,,\\ \partial_y\epsilon(y) &= 2\,[\,\delta(y) - \delta(y - \pi R)\,]\,.\end{aligned}$$

To implement the four-form mechanism within off-shell SUGRA, we introduce a multiplier multiplet defined under a background of $\mathcal{V}_Z$:

$$L_H = (L^{ij},\, \varphi^i,\, E_{\mu\nu\rho},\, H_{\mu\nu\rho\sigma})\,, \tag{2.2}$$

where L^{ij} is a SU(2)-triplet scalar, φ^i is a SU(2)-doublet fermion, $E_{\mu\nu\rho}$ and $H_{\mu\nu\rho\sigma}$ are three and four-form fields, respectively. The off-shell action of this multiplier multiplet was derived in ref. [7] as

$$\begin{aligned}
\mathcal{L}^{\text{bulk}}_{\text{4-form}} &= e(Y^{Sij} - GY^{Zij})L_{ij} - \frac{1}{4!}\epsilon^{\lambda\mu\nu\rho\sigma}\left[\{F_{\lambda\mu}(A^S) - GF_{\lambda\mu}(A^Z)\}\, E_{\nu\rho\sigma} + \frac{1}{2}G\partial_\lambda H_{\mu\nu\rho\sigma}\right], \\
\mathcal{L}^{\text{brane}}_{\text{4-form}} &= (a_0\delta(y) + a_\pi\delta(y-\pi R))\left[\frac{1}{4!}\epsilon^{\mu\nu\rho\sigma y}H_{\mu\nu\rho\sigma} + e_{(4)}\alpha\epsilon^{jk}(i\sigma_3)^i_k L_{ij}\right], \qquad (2.3)
\end{aligned}$$

where ϵ^{jk} is the SU(2)-invariant antisymmetric tensor, $G = \frac{M^S}{M^Z} = \frac{\gamma}{\alpha}$, $e = (-\det(g_{\mu\nu}))^{\frac{1}{2}}$, and $e_{(4)} = (-\det(g_{\underline{\mu\nu}}))^{1/2}$ for the induced 4D metric $g_{\underline{\mu\nu}}$ on the boundaries. Then the equations of motion for $H_{\mu\nu\rho\sigma}$, $E_{\mu\nu\rho}$ and L_{ij} give

$$\begin{aligned}
G &= \frac{\gamma}{\alpha} = \epsilon(y)\,, \\
F_{\mu\nu}(A^S) &= \epsilon(y)F_{\mu\nu}(A^Z)\,, \\
Y^{Sij} &= \epsilon(y)Y^{Zij} + e^{-1}e_{(4)}\alpha\epsilon^{jk}(i\sigma_3)^i_k(\delta(y) - \delta(y-\pi R))\,, \qquad (2.4)
\end{aligned}$$

where the integrability condition of $\partial_y G$ results in $a_0 = -a_\pi = -1/2$. Now using the relations in (2.4), the redundant vector multiplet $\mathcal{V}_S$ can be *replaced* by the central charge vector multiplet $\mathcal{V}_Z$ multiplied by the Z_2-odd factor $\epsilon(y)$. This four-form mechanism provides an elegant way to obtain a locally supersymmetric theory of $\mathcal{V}_I$ $(I = Z, X)$ involving Z_2-odd couplings, e.g., $\xi_{FI}\epsilon(y), k\epsilon(y)$ and $c\epsilon(y)$ in our case, starting from a locally supersymmetric theory of $\mathcal{V}_A$ $(A = Z, X, S)$ and the four-form multiplier multiplet involving only Z_2-even couplings.

Upon ignoring the UV-sensitive higher-dimensional boundary terms, after integrating out the Lagrange multipliers and also the auxiliary fields *other than* Y^{Iij}, V^{ij}_μ, $v_{\mu\nu}$ and $\mathcal{F}^x_i$ (here V^{ij}_μ and $v_{\mu\nu} = -v_{\nu\mu}$ are the auxiliary fields in the Weyl multiplet of 5D conformal SUGRA), we find the following lagrangian density of bosonic fields:

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\text{bulk}} + \mathcal{L}_{\text{brane}} + \mathcal{L}_{\text{aux}}, \\
e^{-1}\mathcal{L}_{\text{bulk}} &= -\frac{1}{2}R - \frac{1}{4}\tilde{a}_{IJ}F^I_{\mu\nu}F^{\mu\nu J} + \frac{1}{2}\tilde{a}_{IJ}\nabla_\mu M^I\nabla^\mu M^J + \frac{1}{8}e^{-1}\tilde{C}_{IJK}\epsilon^{\lambda\mu\nu\rho\sigma}A^I_\lambda F^J_{\mu\nu}F^K_{\rho\sigma} + \\
&\quad + \operatorname{tr}\left[\nabla_\mu\Phi\nabla^\mu\Phi^\dagger - \nabla_\mu\mathcal{A}\nabla^\mu\mathcal{A}^\dagger - V^\dagger_{\mu\,\text{sol}}V^\mu_{\text{sol}} - M^IM^J(\Phi^\dagger t^\dagger_I t_J\Phi - \mathcal{A}^\dagger t^\dagger_I t_J\mathcal{A})\right] - \\
&\quad - \frac{1}{2}\operatorname{tr}\left[\tilde{\mathcal{N}}_{IJ}Y^{I\dagger}Y^J - 4Y^{I\dagger}\left(\mathcal{A}^\dagger t_I\mathcal{A} - \Phi^\dagger t_I\Phi\right)\right], \qquad (2.5) \\
e^{-1}_{(4)}\mathcal{L}_{\text{brane}} &= \Bigg[\frac{1}{2}\xi_{FI}\alpha^2\left(\operatorname{tr}[i\sigma_3Y^X] + e^{-1}e_{(4)}\partial_y\beta\right) + \frac{1}{2}\xi_{FI}\alpha\beta\operatorname{tr}[i\sigma_3Y^Z] - \\
&\quad - 2\alpha\left(3k + \frac{3}{2}k\operatorname{tr}\left[\Phi^\dagger\Phi\right] + c\operatorname{tr}\left[\Phi^\dagger\sigma_3\Phi\sigma_3\right]\right)\Bigg](\delta(y) - \delta(y-\pi R))\,. \qquad (2.6) \\
e^{-1}\mathcal{L}_{\text{aux}} &= -(V_\mu - V_{\mu\,\text{sol}})^{ij}(V^\mu - V^\mu_{\text{sol}})_{ij} + (1 - \alpha^{-2}A^Z_\mu A^{Z\mu})\operatorname{tr}\left[(\mathcal{F} - \mathcal{F}_{\text{sol}})^\dagger(\mathcal{F} - \mathcal{F}_{\text{sol}})\right] \\
&\quad + 2(v - v_{\text{sol}})^{\mu\nu}(v - v_{\text{sol}})_{\mu\nu}\,, \qquad (2.7)
\end{aligned}$$

where

$$
\begin{aligned}
\tilde{\mathcal{N}} &= \mathcal{N}|_{\gamma=\epsilon(y)\alpha} = \tilde{C}_{IJK}M^I M^J M^K = \alpha^3 - \frac{1}{2}\alpha\beta^2 + \frac{1}{2}\xi_{FI}\epsilon(y)\alpha^2\beta, \\
\tilde{a}_{IJ} &= -\frac{1}{2}\frac{\partial^2 \ln \tilde{\mathcal{N}}}{\partial M^I \partial M^J} = \begin{pmatrix} \tilde{a}_{ZZ} & \tilde{a}_{ZX} \\ \tilde{a}_{XZ} & \tilde{a}_{XX} \end{pmatrix}, \\
\tilde{a}_{ZZ} &= \frac{1}{8\tilde{\mathcal{N}}^2}\left(\beta^4 + 2\alpha(6\alpha^3 + 4\xi_{FI}\epsilon(y)\alpha^2\beta - \xi_{FI}\epsilon(y)\beta^3 + \xi_{FI}^2\epsilon^2(y)\alpha\beta^2)\right), \\
\tilde{a}_{XZ} &= \tilde{a}_{ZX} = \frac{1}{8\tilde{\mathcal{N}}^2}\alpha^2\left(-8\alpha\beta + \xi_{FI}\epsilon(y)(2\alpha^2 - \beta^2)\right), \\
\tilde{a}_{XX} &= \frac{1}{8\tilde{\mathcal{N}}^2}\alpha^2\left(2\beta^2 - 2\xi_{FI}\epsilon(y)\alpha\beta + (4 + \xi_{FI}^2\epsilon^2(y))\alpha^2\right), \\
V_{\mu\,\mathrm{sol}} &= -\frac{1}{2}\left(\mathcal{A}^\dagger(\nabla_\mu\mathcal{A}) - (\nabla_\mu\mathcal{A})^\dagger\mathcal{A}\right) + \frac{1}{2}\left(\Phi^\dagger(\nabla_\mu\Phi) - (\nabla_\mu\Phi)^\dagger\Phi\right), \qquad (2.8) \\
\mathcal{F}_{\mathrm{sol}} &= \alpha t_Z \mathcal{A}, \qquad v_{\mu\nu\,\mathrm{sol}} = -\frac{1}{4\tilde{\mathcal{N}}}\tilde{\mathcal{N}}_I F_{\mu\nu}(A^I).
\end{aligned}
$$

Here $\tilde{\mathcal{N}}_I = \frac{\partial\tilde{\mathcal{N}}}{\partial M^I}$, $\tilde{\mathcal{N}}_{IJ} = \frac{\partial^2\tilde{\mathcal{N}}}{\partial M^I\partial M^J}$ for $M^I = (\alpha, \beta)$ $(I = Z, X)$ and

$$
\tilde{\mathcal{N}}^{IJ} = \frac{1}{\beta^2 - \xi_{FI}\epsilon(y)\alpha\beta + (6 + \xi_{FI}^2\epsilon^2(y))\alpha^2}\begin{pmatrix} \alpha & -\beta + \xi_{FI}\epsilon(y)\alpha \\ -\beta + \xi_{FI}\epsilon(y)\alpha & -6\alpha - \xi_{FI}\epsilon(y)\beta \end{pmatrix},
$$

is the inverse matrix of $\tilde{\mathcal{N}}_{IJ}$. Note that after the multiplier multiplet (2.2) is integrated out, the new U(1) generators t_I for hyperscalars are given by

$$
\begin{aligned}
(t_Z,\, t_X)\,\Phi &= (c\epsilon(y),\, q)\, i\sigma_3\Phi, \\
(t_Z,\, t_X)\,\mathcal{A} &= \left(-\frac{3}{2}k\epsilon(y),\, -r\right) i\sigma_3\mathcal{A}.
\end{aligned} \qquad (2.9)
$$

Here we have already integrated out the auxiliary fields other than Y^{Iij}, V_μ^{ij}, $v_{\mu\nu}$ and $\mathcal{F}_i^x$ which we keep at off-shell values since the on-shell values of these auxiliary fields are affected by the boundary supergravity which will be discussed at the end of this section. The complete form of $\mathcal{L}_{\mathrm{aux}}$ involving all auxiliary components can be found in ref. [7].

The 2×2 matrix valued compensator hyperscalar field can be chosen as

$$
\mathcal{A} \equiv \mathbf{1}_2\sqrt{1 + \frac{1}{2}\mathrm{tr}[\Phi^\dagger\Phi]}, \qquad (2.10)
$$

in the unit with the 5D Planck mass $M_5 = 1$, which corresponds to the $\mathrm{SU}(2)_{\mathbf{U}}$ gauge fixing condition in the hypermultiplet compensator formulation of off-shell 5D SUGRA [7]. Also by the dilatation gauge fixing condition the norm function can be fixed as

$$
\tilde{\mathcal{N}} \equiv 1.
$$

Then we have only one physical gauge scalar field ϕ in our system which parameterizes the original scalar fields α and β in $\tilde{\mathcal{N}}$ as

$$\begin{aligned}\alpha &= \frac{\cosh^{2/3}(\phi)}{(1+\xi_{FI}^2\epsilon^2(y)/8)^{1/3}} \\ &= 1+\frac{1}{3}\Big(\phi^2-\frac{1}{8}\xi_{FI}^2\epsilon^2(y)\Big)+\mathcal{O}(\phi^4)\,, \\ \beta &= \frac{\cosh^{2/3}(\phi)[(2+\xi_{FI}^2\epsilon^2(y)/4)^{1/2}\tanh(\phi)+\xi_{FI}\epsilon(y)/2]}{(1+\xi_{FI}^2\epsilon^2(y)/8)^{1/3}} \\ &= \frac{1}{2}\xi_{FI}\epsilon(y)+\sqrt{2}\phi+\mathcal{O}(\phi^3)\,.\end{aligned}$$

The very special manifold spanned by ϕ has the metric

$$\begin{aligned}g_{\phi\phi}(\phi) &= -\frac{1}{2}\frac{\partial^2\ln\tilde{\mathcal{N}}}{\partial M^I\partial M^J}\frac{\partial M^I}{\partial\phi}\frac{\partial M^J}{\partial\phi} \\ &= \frac{1+2\cosh(2\phi)}{3\cosh^2(\phi)} = 1+\frac{1}{3}\phi^2+\mathcal{O}(\phi^4)\,.\end{aligned}$$

Obviously, ϕ is Z_2-odd for Z_2-odd β.

Let us identify the gauged $\mathrm{U}(1)_R$ symmetries of the model. We note that $\mathrm{SU}(2)_R$ representation is labelled by the i,j indices of component fields *after* the $\mathrm{SU}(2)_{\mathbf{U}}$ gauge fixing (2.10). In this setting, gauging the R-symmetry corresponds to making the compensator hypermultiplet to have a nonzero gauge coupling. If the compensator couples to a physical gauge field A_μ^R, the covariant derivative is given by

$$\mathcal{D}_\mu\mathcal{A}_i^x = \partial_\mu\mathcal{A}_i^x-(V_\mu)_{ij}\mathcal{A}^{xj}-(A_\mu^R)_y^x\mathcal{A}_i^y\,,$$

where $(A_\mu^R)_y^x = -(rA_\mu^X+\frac{3}{2}k\epsilon(y)A_\mu^Z)(i\sigma_3)_y^x$ in our case, and V_μ is the auxiliary $\mathrm{SU}(2)_{\mathbf{U}}$ gauge field in the SUGRA (Weyl) multiplet. After the compensator gauge fixing (2.10), the auxiliary $\mathrm{SU}(2)_{\mathbf{U}}$ gauge field is redefined as [14]

$$(V_\mu^N)_j^i = (V_\mu)_j^i-(A_\mu^R)_j^i = (V_\mu)_j^i+\left(rA_\mu^X+\frac{3}{2}k\epsilon(y)A_\mu^Z\right)(i\sigma_3)_j^i\,, \tag{2.11}$$

yielding the R-gauge couplings of all $\mathrm{SU}(2)_{\mathbf{U}}$ non-singlet physical fields, e.g.,

$$\begin{aligned}\mathcal{D}_\nu\psi_\mu^i &= \nabla_\nu\psi_\mu^i-(V_\nu)_j^i\psi_\mu^j \\ &= \nabla_\nu\psi_\mu^i-(V_\nu^N)_j^i\psi_\mu^j+\left(rA_\nu^X+\frac{3}{2}k\epsilon(y)A_\nu^Z\right)(i\sigma_3)_j^i\psi_\mu^j\,,\end{aligned}$$

for the gravitino ψ_μ^i. Therefore when $r\neq 0$, the Z_2-even A_μ^X becomes a $\mathrm{U}(1)_R$ gauge field in the σ_3 direction of $\mathrm{SU}(2)_R$, while for $k\neq 0$ the Z_2-odd graviphoton A_μ^Z becomes a $\mathrm{U}(1)_R$ gauge field again in the σ_3 direction. In this prescription, the Z_2-even (odd) hyperscalar Φ_+ (Φ_-) carries a $\mathrm{U}(1)_X$ charge $q+r$ $(q-r)$, while its fermionic partner carries a $\mathrm{U}(1)_X$ charge q.

The model described by (2.5), (2.6) and (2.7) contain various FI terms which are linear in the auxiliary components $Y^{Z,X}$. For instance, there appear the boundary FI terms of $Y^{X,Z}$ which arise from the ξ_{FI}-term in $\tilde{\mathcal{N}}$ [9]:

$$\frac{1}{2}\xi_{FI}\left[\alpha^2\left(\mathrm{tr}[i\sigma_3 Y^X]+e_4^y\partial_y\beta\right)+\alpha\beta\,\mathrm{tr}[i\sigma_3 Y^Z]\right](\delta(y)-\delta(y-\pi R))\,, \tag{2.12}$$

The first $\mathrm{U}(1)_X$ FI term has been discussed extensively in the literatures [8, 9, 10, 11] together with its physical consequences. (In ref. [11] this FI term is derived in a simpler way by utilizing a superfield approach [15] to 5D conformal supergravity.) As for the second FI term of Y^Z, it involves the product of two Z_2-odd fields, β and $Y^{Z\,(3)}$, and thus depends on how to regulate the behavior of these Z_2-odd fields across the boundary. However, while the first term is $\mathcal{O}(\xi_{FI})$, the second term is $\mathcal{O}(\xi_{FI}^2)$ since $\beta=\mathcal{O}(\xi_{FI})$ near the boundary, thus can be ignored when $\xi_{FI}\ll M_5$ which is the limit that orbifold SUGRA does make sense.

In case with gauged $\mathrm{U}(1)_R$ symmetry, there exist additional FI terms as expected. When $r\neq 0$ and/or $k\neq 0$, the term $2\,\mathrm{tr}\,[Y^{I\dagger}\mathcal{A}^\dagger t_I\mathcal{A}]$ in our bulk lagrangian (2.5) gives a bulk FI term for Y^X and/or Y^Z after the gauge fixing (2.10):

$$2r\,\mathrm{tr}\,[i\sigma_3 Y^X]+3k\epsilon(y)\,\mathrm{tr}\,[i\sigma_3 Y^Z]\,. \tag{2.13}$$

Integrating out Y^Z then leads to a bulk cosmological constant $-3k^2/2\alpha$ with which the total bulk cosmological constant is given by $-k^2(3\alpha^{-1}+9\alpha^2)/2=-6k^2+\mathcal{O}(k^2\phi^2)$. Also as a consequence of (2.4), the bulk FI term of Y^Z appears together with the boundary tension term: $3\alpha k\partial_y\epsilon(y)=(3k+\mathcal{O}(k\phi^2))\partial_y\epsilon(y)$. When $r\neq 0$, there can be additional boundary FI terms of Y^X. To see this, let us briefly review the construction of 4D SUGRA at the boundaries in 5D orbifold SUGRA [7]. The general boundary lagrangian can be written as

$$\mathcal{L}_{\mathrm{N}=1}=\sum_{l=0,\pi}\Lambda_l\delta(y-y_l)\left(-\tfrac{3}{2}[\Sigma\bar{\Sigma}e^{-K^{(l)}(S,\bar{S})/3}]_D+[f_{IJ}^{(l)}(S)W^{I\alpha}W_\alpha^J]_F+[\Sigma^3W^{(l)}(S)]_F\right), \tag{2.14}$$

where $(y_0,y_\pi)=(0,\pi R)$ and $\Lambda_{0,\pi}$ are constants. Σ is the 4D $N=1$ compensator chiral multiplet induced by the 5D compensator hypermultiplet $\mathcal{H}_c$, and S and $W^{I\alpha}$ stand for generic chiral matter and gauge multiplets at the boundaries which come from either bulk fields or pure boundary fields. Here the subscripts D and F represent the D- and F-components, respectively, in the 4D superconformal tensor calculus [7]. Let $\Sigma=(z^0,\chi_R^0,F^0)$ denote the component fields of the $N=1$ compensator superfield. The bosonic components of Σ are given by [7]

$$\begin{aligned}
z^0 &= (\mathcal{A}_+^*)^{2/3},\\
F^0 &= \frac{2}{3}i(\mathcal{A}_+^*)^{2/3}e_4^y\left(V_y^{N\,(1)}+iV_y^{N\,(2)}\right)+\frac{2}{3}\left(1+i\alpha^{-1}e_4^yA_y^Z\right)(\mathcal{A}_+^*)^{-1/3}\left(\tilde{\mathcal{F}}^{(1)}+i\tilde{\mathcal{F}}^{(2)}\right),
\end{aligned}$$

where $\mathcal{A}_+\equiv(\mathcal{A}_{i=2}^{x=2})^*=\sqrt{1+\mathrm{tr}[\Phi^\dagger\Phi]/2}$, V_μ^N is the auxiliary $\mathrm{SU}(2)_\mathbf{U}$ gauge field redefined as (2.11), and $\tilde{\mathcal{F}}\equiv\mathcal{F}-\mathcal{F}_{\mathrm{sol}}=\mathcal{F}-\alpha t_Z\mathcal{A}$. Here we use the notation $V_\mu^N=\sum_{r=1}^3(i\sigma_r)V_\mu^{N\,(r)}$

and $\tilde{\mathcal{F}} = \tilde{\mathcal{F}}^{(0)}\mathbf{1}_2 - \sum_{r=1}^3 (i\sigma_r)\tilde{\mathcal{F}}^{(r)}$. Using the standard tensor calculus, it is straightforward to find the D-component of the real superfield $\Sigma\bar{\Sigma}$:

$$\Sigma\bar{\Sigma}\Big|_D^{\text{bosonic}} = \frac{4}{3}r|z^0|^2(2Y^{X\,(3)} - e_4^y\partial_y\beta) + 2\left(|F^0|^2 + |\hat{\mathcal{D}}_{\underline{\mu}}^{(4)}z^0|^2\right),$$

where

$$\hat{\mathcal{D}}_{\underline{\mu}}^{(4)}z^0 = \frac{2}{3}z_0^{-1/2}\left(\partial_{\underline{\mu}}\mathcal{A}_+^* - i(V_{\underline{\mu}}^{N\,(3)} + e_4^y v_{\underline{\mu}y})\mathcal{A}_+^*\right).$$

Here we omit all the fermionic degrees of freedom and also ignored the terms involving Z_2-odd fields which either vanish at the boundaries or are irrelevant for the following discussion.

As a minimal example of the boundary SUGRA, let us consider the pure 4D SUGRA on the boundaries which corresponds to the case with $K^{(0,\pi)} = f_{IJ}^{(0,\pi)} = W^{(0,\pi)} = 0$ in (2.14). Then the bosonic part of the boundary SUGRA is given by

$$\begin{aligned}\mathcal{L}_{N=1} &= -\frac{3}{2}\left[\Sigma\bar{\Sigma}\right]_D^{\text{bosonic}}(\Lambda_0\delta(y) + \Lambda_\pi\delta(y-\pi R)) \\ &= -e_{(4)}M_{(4)}^2\left[\frac{1}{2}R^{(4)} + 2r(2Y^{X(3)} - e_4^y\partial_y\beta) + 3|z^0|^{-2}\left(|F^0|^2 + |\hat{\mathcal{D}}_{\underline{\mu}}^{(4)}z^0|^2\right)\right]\times \\ &\quad\times(\Lambda_0\delta(y) + \Lambda_\pi\delta(y-\pi R)),\end{aligned} \tag{2.15}$$

where $R^{(4)}$ is the induced Ricci-scalar on the boundaries and

$$M_{(4)}^2 = \left(M_5^3 + \mathrm{tr}\,\frac{[\Phi^\dagger\Phi]}{2}\right)^{2/3}, \tag{2.16}$$

where we recover the 5D Planck mass M_5 which was set as $M_5 = 1$ in the previous discussion. The above boundary SUGRA contains a Fayet-Iliopoulos term

$$-2rM_{(4)}^2\left(2Y^{X\,(3)} - e_4^y\partial_y\beta\right)(\Lambda_0\delta(y) + \Lambda_\pi\delta(y-\pi R)). \tag{2.17}$$

Note that unlike the boundary FI term (2.12) from the ξ_{FI}-term in $\tilde{N}$, these boundary FI terms from $\mathrm{U}(1)_R$ gauging can have independent coefficients at different boundaries, i.e. Λ_0 and Λ_π are independent from each other.

The total action of our $\mathrm{U}(1)_R$-gauged SUGRA including the minimal boundary SUGRA (2.15) is given by

$$\mathcal{L} = \mathcal{L}_{\text{bulk}} + \mathcal{L}_{\text{brane}} + \mathcal{L}_{\text{aux}} + \mathcal{L}_{N=1}. \tag{2.18}$$

In the presence of $\mathcal{L}_{N=1}$, the on-shell values of the auxiliary fields Y^I, V_μ^N, $\tilde{\mathcal{F}} = \mathcal{F} - \alpha t_Z\mathcal{A}$, $v_{\mu\nu}$ are found to be

$$\begin{aligned} Y^I &= \tilde{\mathcal{N}}^{IJ}\tilde{\mathcal{Y}}_J, \qquad V_{\underline{\mu}}^{N\,(1),(2)} = V_{\underline{\mu}\,\text{sol}}^{(1),(2)}, \qquad V_y^N = V_{y\,\text{sol}} + \Delta V_{y\,\text{sol}}, \\ V_{\underline{\mu}}^{N\,(3)} &= \frac{2+\Delta_D}{2+3\Delta_D}V_{\underline{\mu}\text{sol}}^{(3)} - \frac{2\Delta_D}{2+3\Delta_D}e_4^y v_{\underline{\mu}y\,\text{sol}}, \qquad \tilde{\mathcal{F}}^{(0),(3)} = 0, \\ \tilde{\mathcal{F}}^{(1)} &= \frac{\mathcal{A}_+^{-1}\Delta_D}{1 + (\mathcal{A}_+^{-2} - 1)\Delta_D}\frac{e_4^y(V_{y\,\text{sol}}^{(2)} - \alpha^{-1}e_4^y A_y^Z V_{y\,\text{sol}}^{(1)})}{1 - \alpha^{-2}A_\mu^Z A^{Z\mu}},\end{aligned}$$

$$\tilde{\mathcal{F}}^{(2)} = -\frac{\mathcal{A}_+^{-1}\Delta_D}{1+(\mathcal{A}_+^{-2}-1)\Delta_D}\frac{e_4^y(V_{y\,\mathrm{sol}}^{(1)}+\alpha^{-1}e_4^y A_y^Z V_{y\,\mathrm{sol}}^{(2)})}{1-\alpha^{-2}A_\mu^Z A^{Z\mu}},$$
$$v_{\underline{\mu}\underline{\nu}} = v_{\underline{\mu}\underline{\nu}\,\mathrm{sol}}, \qquad v_{\underline{\mu}y} = \frac{2+2\Delta_D}{2+3\Delta_D}v_{\underline{\mu}y\,\mathrm{sol}} - \frac{\Delta_D}{2+3\Delta_D}V_{\underline{\mu}\,\mathrm{sol}}^{(3)}, \tag{2.19}$$

where

$$\tilde{\mathfrak{Y}}_I = 2(\mathcal{A}^\dagger(t_I)\mathcal{A} - \Phi^\dagger(t_I)\Phi) - \frac{1}{2}\xi_{FI}e^{-1}e_{(4)}(i\sigma_3)\alpha^2\delta_I^X(\delta(y)-\delta(y-\pi R)) -$$
$$- 2rM_{(4)}^2(i\sigma_3)\delta_I^X(\Lambda_0\delta(y)+\Lambda_\pi\delta(y-\pi R)),$$
$$\Delta V_{y\,\mathrm{sol}} = \frac{\Delta_D}{1+(\mathcal{A}_+^{-2}-1)\Delta_D}\Big((i\sigma_1)V_{y\,\mathrm{sol}}^{(1)} + (i\sigma_2)V_{y\,\mathrm{sol}}^{(2)}\Big),$$
$$\Delta_D = \frac{2}{3}e^{-1}e_{(4)}M_{(4)}^2(\Lambda_0\delta(y)+\Lambda_\pi\delta(y-\pi R)), \tag{2.20}$$

and $V_{\mu\,\mathrm{sol}} = \sum_{r=1}^3 (i\sigma_3)V_{\mu\,\mathrm{sol}}^{(r)}$ and $v_{\mu\nu\,\mathrm{sol}}$ are defined as eq. (2.8). The part of (2.18) which corresponds to the 5D scalar potential is given by

$$V_{5D} = \mathrm{tr}\left[M^I M^J\{\Phi^\dagger t_I^\dagger t_J\Phi - \mathcal{A}^\dagger t_I^\dagger t_J\mathcal{A}\} - \frac{1}{2}\tilde{\mathcal{N}}^{IJ}\tilde{\mathfrak{Y}}_I^\dagger\tilde{\mathfrak{Y}}_J\right] +$$
$$+ 2e^{-1}e_{(4)}\alpha\left(3k + \frac{3}{2}k\,\mathrm{tr}\left[\Phi^\dagger\Phi\right] + c\,\mathrm{tr}\left[\Phi^\dagger\sigma_3\Phi\sigma_3\right]\right)(\delta(y)-\delta(y-\pi R)).$$

3. 4D Poincaré invariant solutions

In this section, we discuss 4D Poincaré invariant solutions of the $\mathrm{U}(1)_R$-gauged orbifold SUGRA presented in the previous section. We first derive the Killing spinor equations and the energy functional for generic 4D Poincaré invariant metric:

$$ds^2 = e^{2K(y)}\eta_{\underline{\mu}\underline{\nu}}(x)dx^{\underline{\mu}}dx^{\underline{\nu}} - dy^2, \tag{3.1}$$

and then consider some physical implications of the $\mathrm{U}(1)_R$ FI terms associated with a $\mathrm{U}(1)_R$ charge $r \neq 0$.

3.1 Killing conditions and 4D energy functional

Applying the local SUSY transformations of the gravitinos, gauginos, and the compensator and physical hyperinos [7], we find the corresponding Killing spinor conditions:

$$\kappa \equiv \partial_y K - \frac{1}{3}M^I\tilde{\mathfrak{Y}}_I(i\sigma_3)^\dagger = 0,$$
$$G^I \equiv \partial_y M^I - 2\left(\tilde{\mathcal{N}}^{IJ} - \frac{1}{6}M^I M^J\right)\tilde{\mathfrak{Y}}_J = 0, \tag{3.2}$$
$$F \equiv \partial_y\Phi - \Phi(V_y^N)^\dagger - M^I(gt_I)\Phi(i\sigma_3)^\dagger + \frac{1}{2}\Phi M^I\tilde{\mathfrak{Y}}_I(i\sigma_3)^\dagger = 0,$$
$$\tilde{F} \equiv \partial_y\mathcal{A} - \mathcal{A}(V_y^N)^\dagger - \tilde{\mathcal{F}}\Big((i\sigma_3)^\dagger + \alpha^{-1}A_y^Z\Big) - M^I(gt_I)\mathcal{A}(i\sigma_3)^\dagger + \frac{1}{2}\mathcal{A}M^I\tilde{\mathfrak{Y}}_I(i\sigma_3)^\dagger = 0,$$

where

$$\tilde{\mathcal{Y}}_Z = -2\epsilon(y)\Big(\frac{3}{2}k\mathcal{A}^\dagger(i\sigma_3)\mathcal{A} + c\Phi^\dagger(i\sigma_3)\Phi\Big),$$

$$\tilde{\mathcal{Y}}_X = -2\Big(r\mathcal{A}^\dagger(i\sigma_3)\mathcal{A} + q\Phi^\dagger(i\sigma_3)\Phi\Big) - \frac{1}{2}g\xi_{FI}\alpha^2(i\sigma_3)(\delta(y)-\delta(y-\pi R))$$
$$-2rM_{(4)}^2(i\sigma_3)(\Lambda_0\delta(y)+\Lambda_\pi\delta(y-\pi R))\,,$$

and $M_{(4)}^2$ defined in eq. (2.16). When all the above Killing conditions are satisfied, there can be unbroken 4D $N=1$ supersymmetry with the corresponding Killing spinor

$$\epsilon_+(y) = \exp\left[\frac{1}{2}(K(y)-K(0))\mathbf{1}_2 + \int_0^y dz\, V_y^N(z)\right]\epsilon_+(0)\,,$$

where $\epsilon_+ = (\epsilon_R^{i=1}, \epsilon_L^{i=2})$ and ϵ^i is the 5D supersymmetry transformation parameter.

The 4D energy density of a 4D Poincaré invariant configuration is given by

$$E = \int dy\, e^{4K}\,\mathrm{tr}\left[\frac{1}{4}\tilde{a}_{IJ}G^{I\dagger}G^J - 3|\kappa|^2 + |F|^2 - |\tilde{F}|^2 + |\Delta F|^2 - |\Delta\tilde{F}|^2 - \right.$$
$$\left. -(F^\dagger\Delta F - \tilde{F}^\dagger\Delta\tilde{F} + \text{h.c.}) - 2|V_{y\,\mathrm{sol}}|^2 - \frac{1}{2}M^I V_{y\,\mathrm{sol}}[\mathcal{Y}_I,\, i\sigma_3] + \Delta\mathcal{E}\right],$$

where

$$\Delta F = -\Phi\Delta V_{y\,\mathrm{sol}}^\dagger\,, \qquad \Delta\tilde{F} = -\mathcal{A}\Delta V_{y\,\mathrm{sol}}^\dagger - \tilde{\mathcal{F}}\Big((i\sigma_3)^\dagger + \alpha^{-1}A_y^Z\mathbf{1}_2\Big)\,,$$
$$\Delta\mathcal{E} = \frac{\Delta_D}{1+(\mathcal{A}_+^{-2}-1)\Delta_D}\Big((V_{y\,\mathrm{sol}}^{(1)})^2 + (V_{y\,\mathrm{sol}}^{(2)})^2\Big)\mathbf{1}_2\,,$$

for $V_{y\,\mathrm{sol}}$, $\tilde{\mathcal{F}}$ and $\Delta V_{y\,\mathrm{sol}}$ given by eqs. (2.8), (2.19) and (2.20). To arrive at this form of 4D energy density starting from the 5D action (2.18), we have truncated the UV sensitive higher-order boundary operators of $\mathcal{O}(\lambda\xi_{FI}^2)$, $\mathcal{O}(\lambda\Lambda_{0,\pi}^2)$ $(\lambda = (\xi_{FI}, \Lambda_{0,\pi}, k, c))$ whose precise value depend on how to regulate the Z_2-odd fields at the boundaries. The above form of the 4D energy density indicates that a field configuration satisfying the Killing conditions:

$$\kappa = G^I = F = \tilde{F} = 0\,,$$

as well as the stationary conditions:

$$V_{y\,\mathrm{sol}} = 0\,, \qquad [\mathcal{Y}_I,\, \sigma_3] = 0\,,$$

corresponds to a supersymmetric solution with vanishing vacuum energy. A simple solution of the above stationary conditions is

$$\Phi = \begin{pmatrix} v(y) & 0 \\ 0 & v(y)\end{pmatrix}, \tag{3.3}$$

where v is a real function of y.

If $\xi_{FI} = 0$, the gaugino Killing conditions are simplified as

$$G^I \equiv \frac{\partial M^I}{\partial \phi} D = \frac{\partial M^I}{\partial \phi} \left[\partial_y \phi + g^{\phi\phi} \frac{\partial M^I}{\partial \phi} \tilde{\mathcal{Y}}_I (i\sigma_3)^\dagger \right] = 0 \,.$$

Since the physical implications of the FI coefficient ξ_{FI} have been studied extensively before [8, 9, 10, 11], here we restrict ourselves to the case of $\xi_{FI} = 0$ but $r \neq 0$, while leaving the more general case with $\xi_{FI} \neq 0$ and $r \neq 0$ for future work. When $\xi_{FI} = 0$, the Killing conditions and the form of the 4D energy density can be simplified under the ansatz (3.3) for the physical hyperscalar field. In this situation, we have

$$M^I \tilde{\mathcal{Y}}_I (i\sigma_3)^\dagger = \mathcal{P} \mathbf{1}_2 \,, \qquad \frac{\partial M^I}{\partial \phi} \tilde{\mathcal{Y}}_I (i\sigma_3)^\dagger = \partial_\phi (\mathcal{P} + \Xi) \mathbf{1}_2 \,,$$

where

$$\begin{aligned} \mathcal{P} &\equiv -2 \left[\left(\frac{3}{2} k \epsilon(y) \alpha + r\beta \right) + v^2 \left(\left(\frac{3}{2} k + c \right) \epsilon(y) \alpha + (r+q)\beta \right) \right] , \\ \Xi &\equiv -2r M_{(4)}^2 \beta \Big(\Lambda_0 \delta(y) + \Lambda_\pi \delta(y - \pi R) \Big) \,, \end{aligned}$$

where we again ignore the UV-sensitive higher-dimensional boundary operators. We then find the Killing parameters:

$$\begin{aligned} \kappa &= \partial_y K - \frac{1}{3} \mathcal{P} \,, \\ D &= \partial_y \phi + g^{\phi\phi} \partial_\phi (\mathcal{P} + \Xi) \,, \\ F &= \partial_y v - v \left(q\beta + c\epsilon(y)\alpha - \frac{1}{2} \mathcal{P} \right) , \\ \tilde{F} &= \frac{v}{\sqrt{1+v^2}} F \,, \end{aligned}$$

and also the 4D energy density:

$$E = \int dy\, e^{4K} \left[\frac{1}{2} g_{\phi\phi} D^2 + \frac{2}{1+v^2} F^2 - 6\kappa^2 \right] ,$$

which tells us that the field configuration satisfying

$$\kappa = D = F = 0 \,, \tag{3.4}$$

corresponds to a supersymmetric vacuum solution of the theory.

3.2 Vacuum solutions with $\mathrm{U}(1)_R$ FI terms

In this subsection, we discuss some aspects of the vacuum solution in gauged $\mathrm{U}(1)_R$ SUGRA on S^1/Z_2. Before going to the main analysis, we briefly discuss the condition for the $\mathrm{U}(1)_R$ anomaly cancellation. To be complete, let us introduce boundary chiral multiplets

$$S_0^a = (z_0^a, \chi_0^a, f_0^a) \,, \qquad S_\pi^\alpha = (z_\pi^\alpha, \chi_\pi^\alpha, f_\pi^\alpha) \,,$$

confined at $y = 0$ and $y = \pi R$, respectively, and let q_0^a and q_π^α denote the $\mathrm{U}(1)_X$ charge of their scalar components z_0^a and z_π^α, respectively. Here $\chi_0^a, \chi_\pi^\alpha$ and f_0^a, f_π^α denote the chiral fermion and the complex auxiliary components of S_0^a, S_π^α, respectively. Then the $\mathrm{U}(1)_R^3$ and $\mathrm{U}(1)_R$-gravity-gravity anomaly cancellation conditions are given by

$$
\begin{aligned}
(3+1)r^3 + \sum_{\text{gaugino}} r^3 + \sum_{\text{bulk}} q^3 + \sum_a (q_0^a - r)^3 + \sum_\alpha (q_\pi^\alpha - r)^3 &= 0\,, \\
(-21+1)r + \sum_{\text{gaugino}} r + \sum_{\text{bulk}} q + \sum_a (q_0^a - r) + \sum_\alpha (q_\pi^\alpha - r) &= 0\,,
\end{aligned}
\tag{3.5}
$$

where the first terms represent the contributions from the gravitino and radino zero modes.

To see the effects of Z_2-even $\mathrm{U}(1)_R$-gauging, i.e., of $r \neq 0$, let us first consider the simplest situation that $\xi_{FI} = \Lambda_0 = \Lambda_\pi = k = c = 0$ and there is no $\mathrm{U}(1)_X$-charged boundary matter fields. In this case, a unique solution of the Killing conditions (3.4) is given by $K = \phi = 0$ and a *constant* hyperscalar VEV

$$
v = v_0 \equiv \pm\sqrt{-\frac{r}{r+q}}\,.
$$

Such hyperscalar VEV is allowed as long as $q/r < -1$, which is in fact required in order for the anomaly cancellation condition (3.5) to be satisfied.

If one introduces boundary $\mathrm{U}(1)_R$ FI terms ($\Lambda_{0,\pi} \neq 0$) into the above model, the supersymmetric vacuum solution is deformed as follows. The corresponding Killing spinor conditions are given by

$$
\begin{aligned}
\partial_y K &= -\frac{2\sqrt{2}}{3}\Big(r + (r+q)v^2\Big)\cosh^{2/3}(\phi)\tanh(\phi)\,, \\
\partial_y \phi &= 2I(\phi)\left[r + (r+q)v^2 + rM_{(4)}^2(v)(\Lambda_0\delta(y) + \Lambda_\pi\delta(y-\pi R))\right], \\
\partial_y v &= \sqrt{2}(r+q)(1+v^2)v\cosh^{2/3}(\phi)\tanh(\phi)\,,
\end{aligned}
\tag{3.6}
$$

where

$$
I(\phi) = \frac{\sqrt{2}\cosh^{2/3}(\phi)\,(2+\cosh(2\phi))}{1+2\cosh(2\phi)}\,.
$$

Obviously, the existence of $\Lambda_0\delta(y) + \Lambda_\pi\delta(y-\pi R)$ in the gaugino Killing condition enforces $\phi(y)$ to have a non-trivial y-dependence, and thus $v(y)$ also. In the limit that $|\phi(y)| \ll 1$ over the entire orbifold, which would be the case if $|r|, |q| \ll 1$ (in the unit with $M_5 = 1$), the gaugino and hyperino Killing conditions can be approximated as

$$
\begin{aligned}
\partial_y \phi &\simeq 2\sqrt{2}\left[r + (r+q)v^2 + rM_{(4)}^2(v)(\Lambda_0\delta(y) + \Lambda_\pi\delta(y-\pi R))\right], \\
\partial_y v &\simeq \sqrt{2}(r+q)v(1+v^2)\,\phi\,.
\end{aligned}
\tag{3.7}
$$

Then at leading order in $\delta v \equiv v - v_0$ ($v_0 = \pm\sqrt{-r/(r+q)}$) which is presumed to be a small vacuum deformation, we find

$$
\begin{aligned}
v &= v_0 + (Ae^{\omega y} + Be^{-\omega y})\,, \\
\phi &= -2\frac{|rv_0|}{rv_0}\sqrt{1+\frac{r}{q}}\,\left(Ae^{\omega y} - Be^{-\omega y}\right)\,,
\end{aligned}
\tag{3.8}
$$

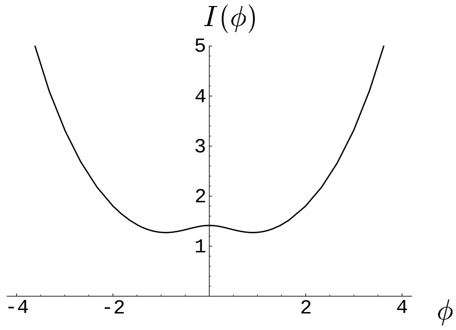

Figure 1: Numerical value of $I(\phi)$ for $|\phi| \leq 4$.

for $0 < y < \pi R$, where $\omega = \sqrt{-8rq}$ and

$$A = \frac{\sqrt{2}|\,rv_0|}{2v_0}\left(\frac{q}{r+q}\right)^{7/6}\left(\frac{\Lambda_0 + \Lambda_\pi e^{\omega\pi R}}{e^{2\omega\pi R}-1}\right),$$
$$B = \frac{\sqrt{2}|\,rv_0|}{2v_0}\left(\frac{q}{r+q}\right)^{7/6}\left(\frac{\Lambda_0 + \Lambda_\pi e^{-\omega\pi R}}{1-e^{-2\omega\pi R}}\right).$$

The above relations between the integration constants A, B and the orbifold radius R show that once there exists a dynamics to determine one (or both) of the boundary values of $v(y)$, e.g., the boundary superpotentials of Φ which would determine $v(0) = v_0 + A + B$ and/or $v(\pi R) = v_0 + Ae^{\omega\pi R} + Be^{-\omega\pi R}$, one might be able to fix R as well as to break $N = 1$ SUSY through the combined effects of the $\mathrm{U}(1)_R$ FI terms and the boundary superpotentials. Note that both $\phi(y)$ and $\delta v(y)$ are small over the entire orbifold when $|\,r|, |q| \ll 1$, justifying the use of the approximate Killing conditions (3.7).

As another example, we can consider the case that the D-flat condition, i.e., the gaugino Killing condition, is satisfied through the nonzero VEVs of the boundary chiral multiplets S_0^a and/or S_π^α, while the bulk hyperscalars have vanishing VEVs. For simplicity, let us again set $\xi_{FI} = k = c = 0$, while keeping r and $\Lambda_{0,\pi}$ to be nonzero, and assume the minimal form of the boundary Kähler potentials: $K^{(0)}(S_0, \bar{S}_0) = \sum_a \bar{S}_0^a S_0^a$ and $K^{(\pi)}(S_\pi, \bar{S}_\pi) = \sum_\alpha \bar{S}_\pi^\alpha S_\pi^\alpha$. Then the gravitino and gaugino Killing conditions are given by

$$\partial_y K = -\frac{2\sqrt{2}}{3} r \cosh^{2/3}(\phi)\tanh(\phi)\,,$$
$$\partial_y \phi = 2I(\phi)\left[\,r + \lambda_0\delta(y) + \lambda_\pi\delta(y-\pi R)\right]\,, \tag{3.9}$$

where $\lambda_0 = (r + \sum_a q_0^a |z_0^a|^2)\Lambda_0$ and $\lambda_\pi = (r + \sum_\alpha q_\pi^\alpha |z_\pi^\alpha|^2)\Lambda_\pi$. In the limit that $|\,\phi(y)| \ll 1$ over the entire orbifold, this D-flat condition leads to

$$\phi \simeq 2\sqrt{2}ry + \sqrt{2}\lambda_0,$$
$$e^{2K} \simeq \exp\left(-\frac{8}{3}r^2y^2 - \frac{8}{3}\lambda_0 ry\right)\,, \tag{3.10}$$

for $0 < y < \pi R$. Although derived under the condition that $|\,\phi(y)| \ll 1$ for $0 < y < \pi R$, the above approximate solutions are valid as long as $|\,\phi(y)| \lesssim 1$ for which $I(\phi)$ is approximately a constant as can be seen from figure 1. (If $|\phi| \gtrsim 1$ near the boundary, the resulting boundary fluctuations of $\beta(\phi)$ would be too large to be described by orbifold field theory.) In this case, the integrability condition $\oint dy\, \partial_y \phi = 0$ determines the orbifold radius (for given values of $\lambda_{0,\pi}$) as

$$2\pi R \simeq -\frac{\lambda_0 + \lambda_\pi}{r}\,.$$

Since $|\lambda_0 + \lambda_\pi| \lesssim 1$ in order for the orbifold field theory description to be valid, the above relation indicates that $2\pi R \lesssim 1/|r|$ for which the warp factor e^{2K} is approximately a constant. It also implies that once there exists a dynamics to determine the VEVs of the boundary scalar fields z_0^a and z_π^α, e.g., the boundary superpotentials, one might be able to fix R through the combined effects of the $\mathrm{U}(1)_R$ FI terms and the boundary superpotentials.

However the nontrivial profile of ϕ due to the Z_2-even $\mathrm{U}(1)_R$ gauging can significantly alter the shape of the zero-mode wavefunction $\Phi^{(0)}(y)$ of a $\mathrm{U}(1)_X$-charged hypermultiplet. To see this, let us consider a matter hypermultiplet with nonzero hyperino $\mathrm{U}(1)_X$ charge q ($qr > 0$) and vanishing VEV. The corresponding zero mode obeys

$$\partial_y \Phi^{(0)}(y) - m(y)\Phi^{(0)}(y) = 0\,,$$

where

$$m(y) = (3k/2 + c)\epsilon(y)\alpha(\langle\phi(y)\rangle) + (q + r)\beta(\langle\phi(y)\rangle)\,,$$

in most general situation. For $\xi_{FI} = k = c = 0$ which leads to $\phi(y)$ given by (3.10), one easily finds

$$\Phi^{(0)}(y) \simeq \Phi^{(0)}(0)\exp\left[2(q + r)(ry^2 + \lambda_0 y)\right]\,. \tag{3.11}$$

For $\pi R|q(\lambda_0 - \lambda_\pi)| \gtrsim 1$, this matter zero mode would be quasi-localized at one of the orbifold boundaries as

$$\frac{\Phi^{(0)}(\pi R)}{\Phi^{(0)}(0)} \simeq \exp\left[(q + r)\pi R(\lambda_0 - \lambda_\pi)\right]\,.$$

If $|q\pi R|$ is large enough, the resulting quasi-localization of matter zero modes can generate small Yukawa couplings in a natural manner as proposed in ref. [12]. In figure 2, we show the profiles of $\phi(y)$ and the corresponding hypermultiplet zero mode $\Phi^{(0)}$ in some cases with $k = \lambda_0 = 0$ and $\lambda_\pi \neq 0$. In this analysis, we consider also the case that the bare hypermultiplet kink mass $c\epsilon(y)$ is non-vanishing. Note that $\phi(y)$ takes a linear profile yielding the gaussian profile of the matter zero mode wavefunction. The wave-function suppression at $y = 0$ caused by the gauged $\mathrm{U}(1)_R$ FI terms can be as important as the effect of the bare kink mass $c\epsilon(y)$. As can be noticed from figure 2, one can have a stronger suppression for smaller r since then a larger orbifold radius is allowed. We remark that if both λ_0 and λ_π are non-vanishing, the location of the minimum of the wave-function can be shifted. The extreme case would be $\lambda_0 = \lambda_\pi$ for which the minimum is located at $y = \pi R/2$.

Figure 3 reveals the profiles of ϕ and $\Phi^{(0)}$ for some cases with nonvanishing k. Here ϕ has different profile for different sign of k, figure 3(a) for $rk < 0$ and (b) for $rk > 0$. Due to the effect of nonvanishing k, the orbifold radius in figure 3(a) is larger than the

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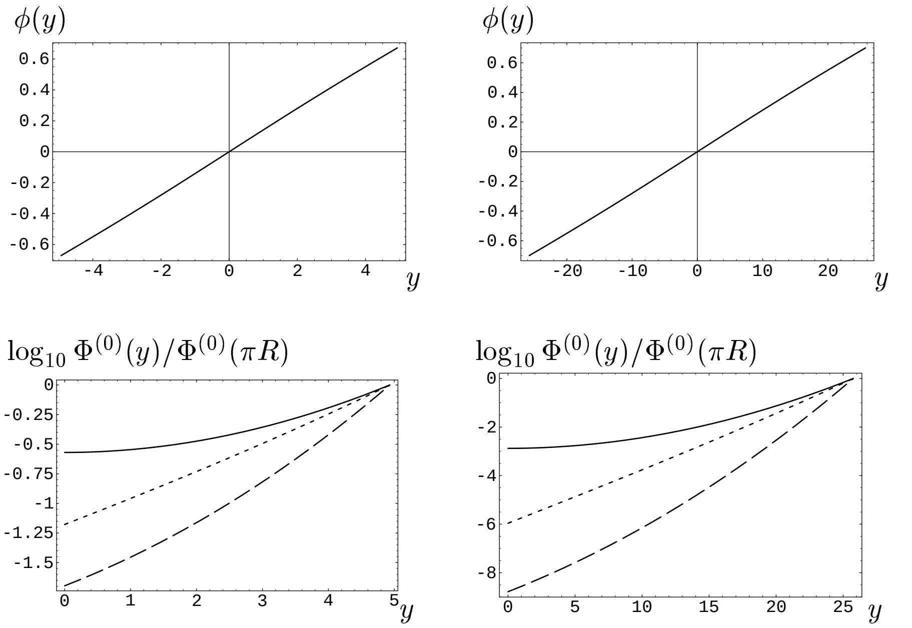


(a) $r = 0.05,\ k = 0$ (b) $r = 0.01,\ k = 0$

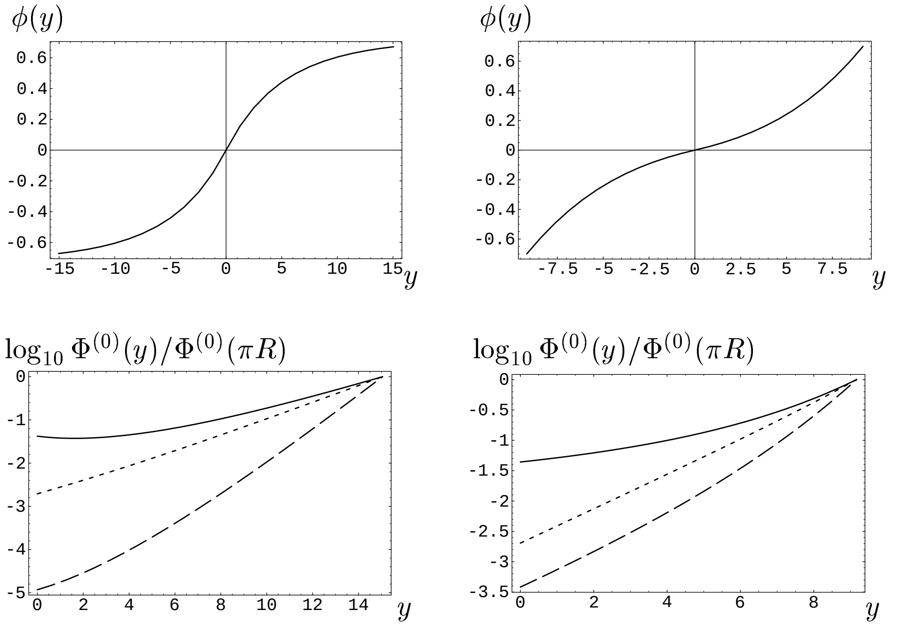
Figure 2: The profiles of ϕ and the matter zero mode $\Phi^{(0)}$ for some cases with $k = 0$ and $\lambda_0 = 0$. Here we choose $\lambda_\pi = (r-1)/2$. For the matter zero mode profile, the solid-, dotted- and dashed-curves represent the case with $(q, c) = (0.5, 0)$, $(0, 0.5)$ and $(0.5, 0.5)$, respectively. All parameters are given in the unit with $M_5 = 1$, and all the curves are shown within $|y| \le \pi R$.

one in figure 2(a), while the radius in figure 3(b) is smaller than the one in figure 2(b). Consequently the wave-function suppression becomes more (less) significant for $rk < 0$ ($rk > 0$) compared to the case with $k = 0$.

4. Conclusion

In this paper, we have studied a 5D gauged $\mathrm{U}(1)_R$ supergravity on S^1/Z_2 in which both a Z_2-even U(1) gauge field and the Z_2-odd graviphoton take part in the $\mathrm{U}(1)_R$ gauging. Based on the off-shell 5D supergravity of refs. [7], we examined the structure of Fayet-Iliopoulos (FI) terms allowed by such theory. As expected, introducing a Z_2-even $\mathrm{U}(1)_R$ gauging accompanies new bulk and boundary FI terms in addition to the known integrable boundary FI term which could be present in the absence of any gauged $\mathrm{U}(1)_R$ symmetry. The new (non-integrable) boundary FI terms originate from the $N = 1$ boundary supergravity, and thus are free from the bulk supergravity structure in contrast to the integrable boundary FI term which is determined by the bulk structure of 5D supergravity [8, 9, 10, 11].

We have examined some physical consequences of the Z_2-even $\mathrm{U}(1)_R$ gauging in several simple cases. It is noted that the FI terms of gauged Z_2-even $\mathrm{U}(1)_R$ can lead to an interesting deformation of vacuum structure which can affect the quasi-localization of the matter zero modes in extra dimension and also the SUSY breaking and radion stabilization. Thus the 5D gauged $\mathrm{U}(1)_R$ supergravity on orbifold has a rich theoretical structure which



(a) $r = 0.05$, $k = -0.1$ (b) $r = 0.01$, $k = 0.1$

Figure 3: The profiles of ϕ and $\Phi^{(0)}$ for $r, k \neq 0$, $\lambda_0 = 0$ and $\lambda_\pi = (r-1)/2$. Again the solid-, dotted- and dashed-curves represent the case $(q, c) = (0.5, 0)$, $(0, 0.5)$ and $(0.5, 0.5)$, respectively. Note that $K \simeq -ky$ in this supersymmetric solution.

may be useful for understanding some problems in particle physics such as the Yukawa hierarchy [12] and/or the supersymmetry breaking. When one tries to construct a realistic particle physics model within gauged $U(1)_R$ supergravity, the most severe constraint comes from the anomaly cancellation condition [3, 16]. In some cases the Green-Schwarz mechanism might be necessary to cancel the anomaly, which may introduce another type of FI term into the theory [17]. These issues will be studied in future works.

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